Transcomputation to infinity and beyond!

Dr James Anderson FBCS CITP CSci
安德生

Topics

- A complex story made simple
- Learning the lessons of history - repeating success!
- How to divide by zero
- The future

History of Complex Numbers

Complex Numbers

- What is the square root of a negative number?
- $\sqrt{-4} = ?$

Complex Numbers

- Define $i = j = \sqrt{-1}$

Complex Numbers

- Now $(i2)^2 = i^2 2^2 = -1 \times 4 = -4$
- So $\sqrt{-4} = i2$
- But people argued about this for 400 years!

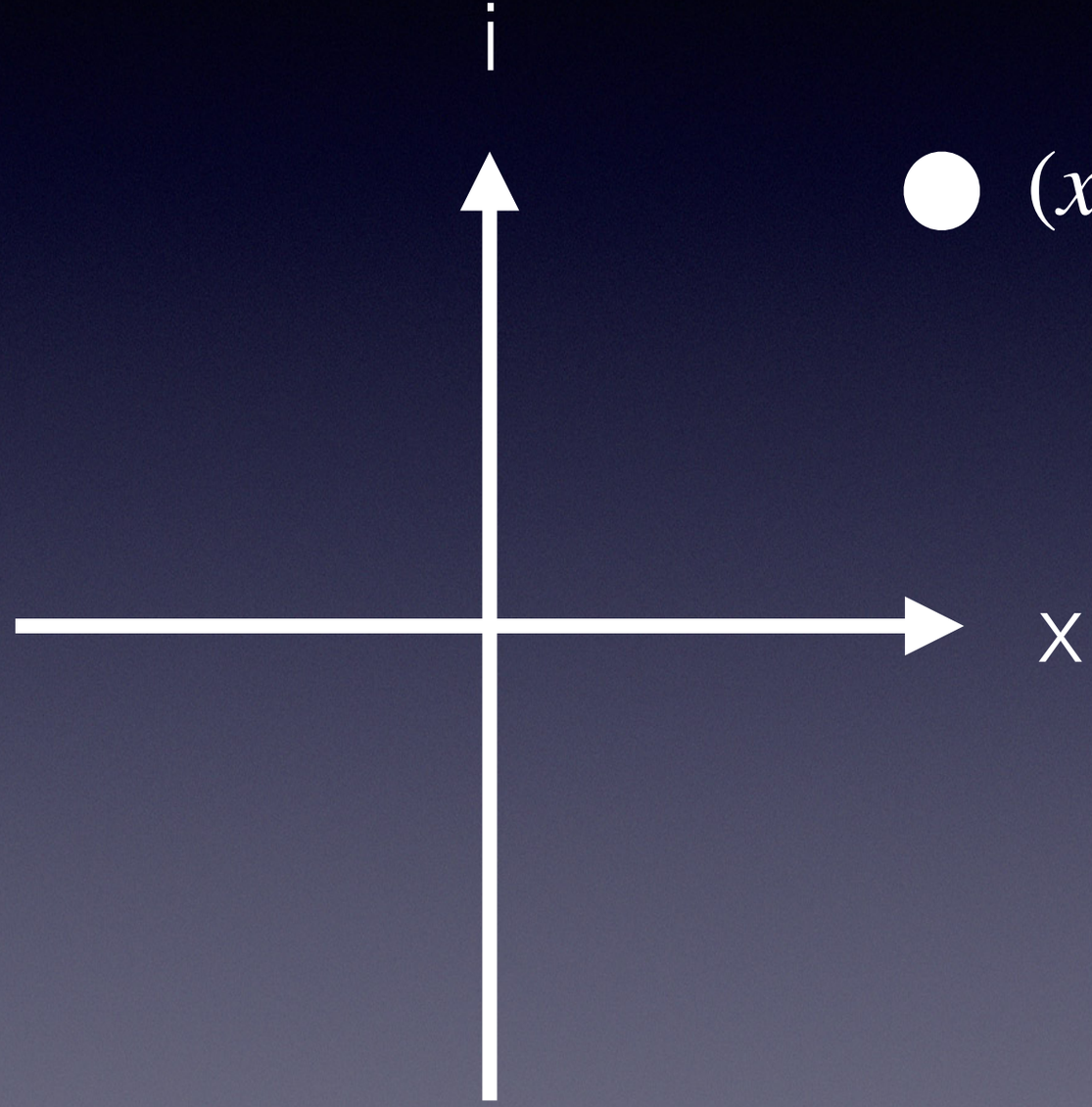
Complex Numbers

- Complex numbers were constructed from real numbers

- So complex arithmetic is consistent if real arithmetic is

- So complex arithmetic cannot be disproved in its own terms, only real arithmetic can be disproved

Complex Numbers



Complex Numbers

- Really useful in physics and signal processing
- People came to love complex numbers!

How to Divide by Zero

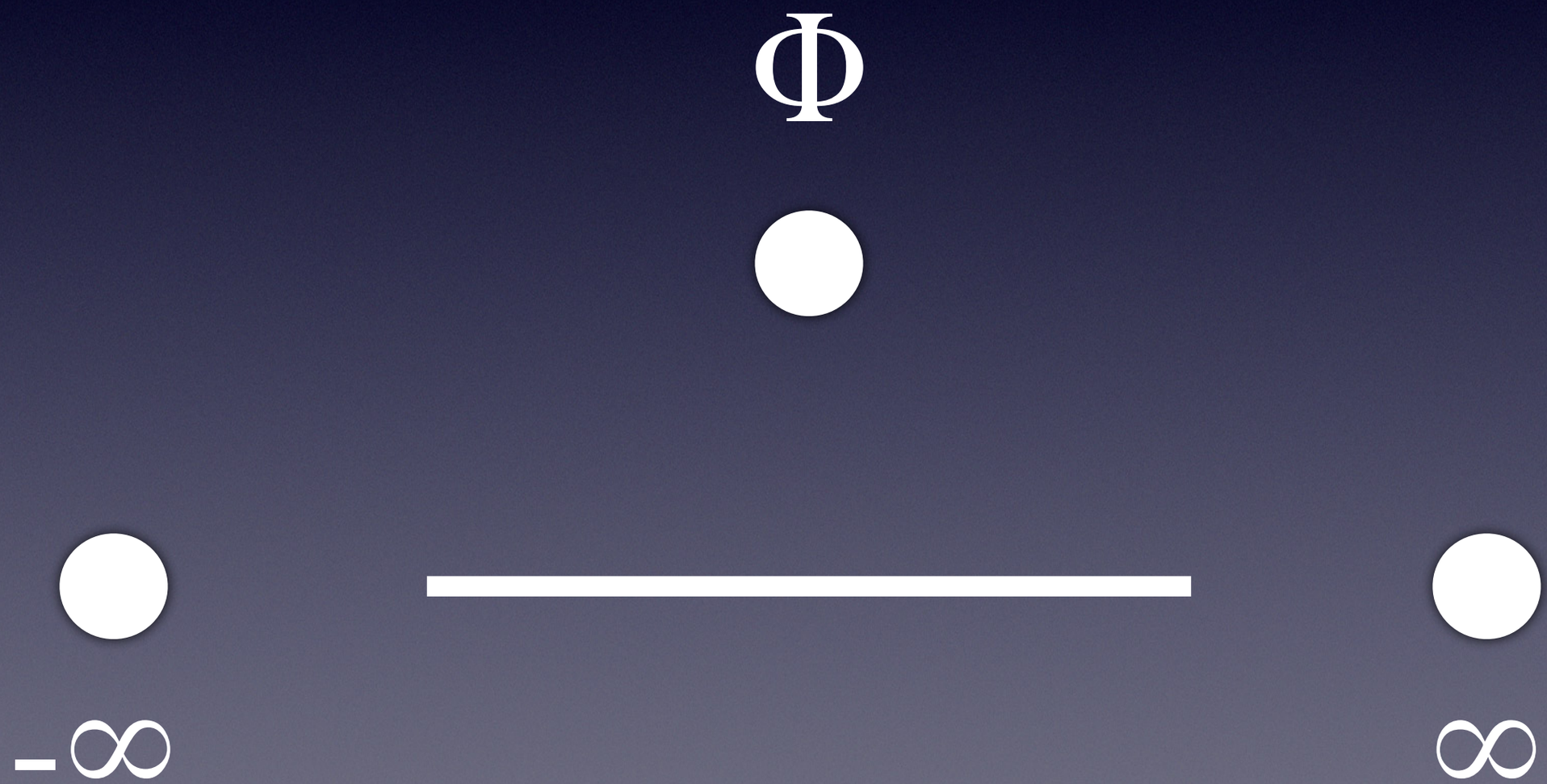
Consistency

- Transreal arithmetic proved consistent by machine proof (2006)

- Transreal (2016) and transcomplex (2014) arithmetic proved consistent by construction from, respectively, the real and complex numbers

- So transreal and transcomplex arithmetic are consistent if real arithmetic is and can only be disproved if real arithmetic is

Transreal-Number Line



Transreal Numbers

Transreal numbers, t, are proper fractions of real numbers, with a non-negative denominator, d, and a numerator, n, that is one of -1, 0, 1 when d = 0

$$t = \frac{n}{d}$$

With k a positive constant:

$$-\infty = \frac{-k}{0} = \frac{-1}{0} \qquad \Phi = \frac{0}{0} \qquad \infty = \frac{k}{0} = \frac{1}{0}$$

Negative Denominators

An improper fraction may have a negative denominator (-k) which must be made positive *before* any transarithmetical operator is applied

$$\frac{n}{-k} = \frac{-n}{-(-k)} = \frac{-1 \times n}{-1 \times (-k)} = \frac{-n}{k}$$

Multiplication

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Division

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

Addition of Two Infinities

$$\infty+\infty=\frac{1}{0}+\frac{1}{0}=\frac{1+1}{0}=\frac{2}{0}=\frac{1}{0}=\infty$$

$$\infty+(-\infty)=\frac{1}{0}+\frac{-1}{0}=\frac{1-1}{0}=\frac{0}{0}=\Phi$$

$$-\infty+\infty=\frac{-1}{0}+\frac{1}{0}=\frac{-1+1}{0}=\frac{0}{0}=\Phi$$

$$-\infty+(-\infty)=\frac{-1}{0}+\frac{-1}{0}=\frac{-1+(-1)}{0}=\frac{-2}{0}=\frac{-1}{0}=-\infty$$

General Addition

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

Subtraction

$$\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \frac{-c}{d}$$

Associativity

$$a+(b+c)=(a+b)+c$$

$$a\times(b\times c)=(a\times b)\times c$$

Commutativity

$$a + b = b + a$$

$$a \times b = b \times a$$

Partial Distributivity

$$a(b+c) = ab + ac$$

When $a \neq \pm\infty$ or

$bc > 0$ or

$(b+c)/0 = \Phi$

Comparison

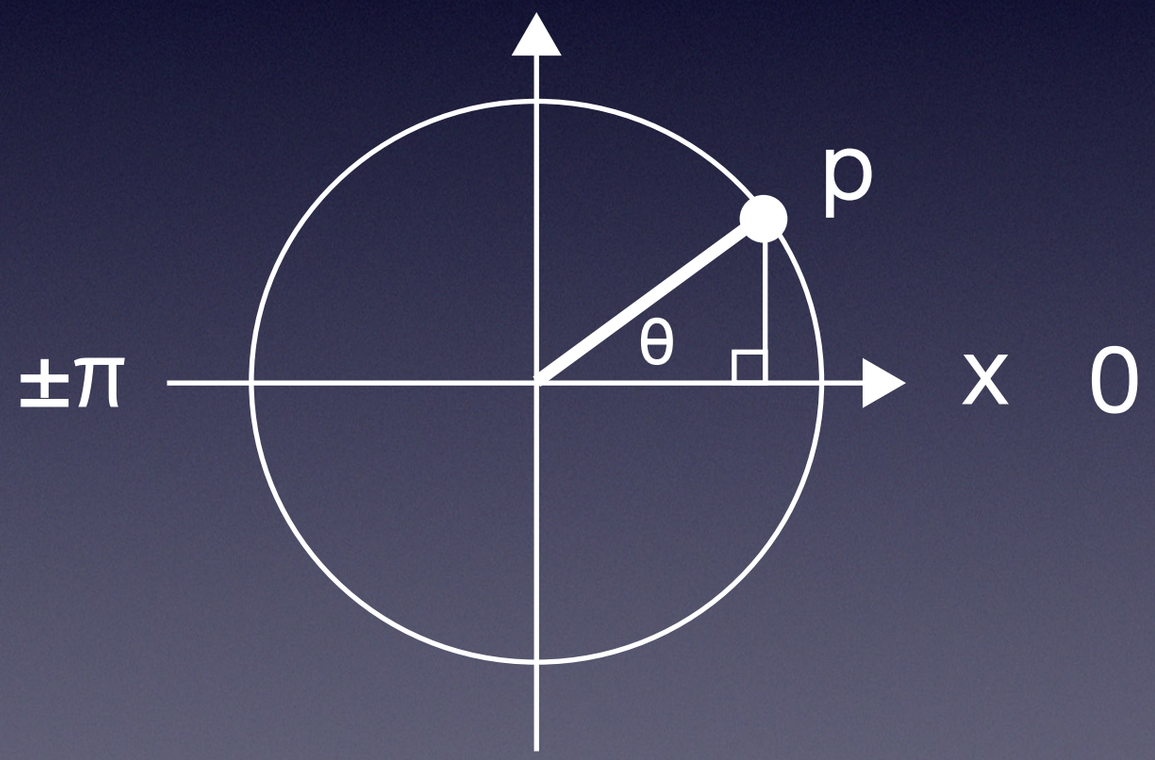
- Mathematics checks for division by zero and, if found, it fails
- Transmathematics checks for division by zero and always succeeds

The Future

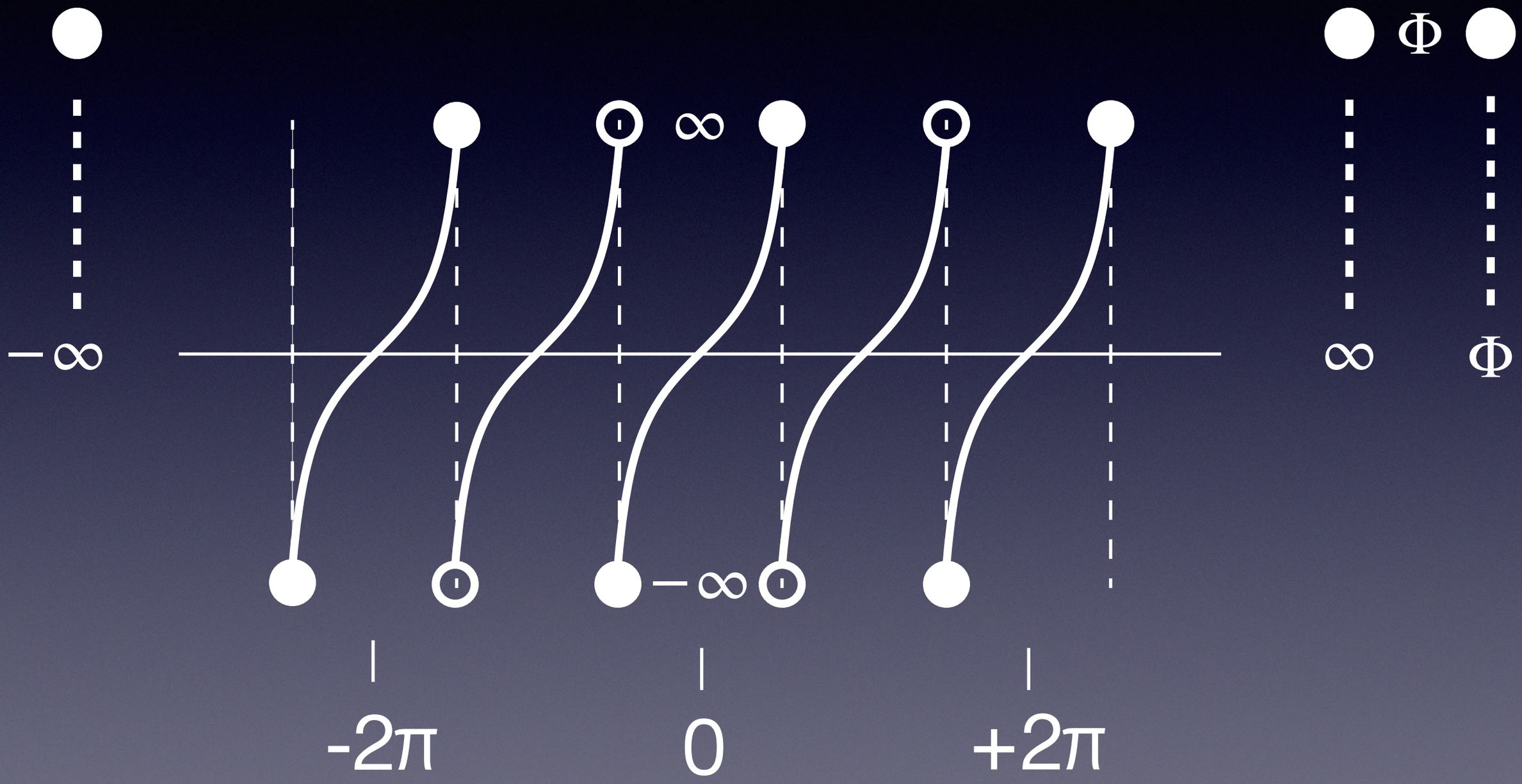
The Future

- We think the whole of mathematics extends to a total (trans) system
- We know Newton's Laws of Motion extend to transmathematics
- We know that transmathematical computer hardware and software can be built

Tangent



Tangent



Nullity Force

- There is no component of nullity on the real-number-line, extended with positive and negative infinity, so nullity forces have no, i.e. zero, effect on the extended-real universe where we live

Newton’s Law 1

- A mass is accelerated only by a positive or negative force, not a zero or nullity force

Newton's Law 2

- $F = ma$ when $0 < m < \infty$ and a is transreal
- $a = F / m$ when $0 < m < \infty$ and F is transreal
- $m = F / a$ when a, F are transreal. When the computed mass is real, it is determined. When the computed mass is nullity, the true, finite, mass is hidden (but can be discovered via gravitation)

Newton’s Law 3

- To any action, F, there is always an opposite and equal reaction, -F

Von Neumann Computer

- Lies about the physics of the universe: data can be moved any distance in unit time!
- 2 GHz core stalls 90% of the time until the lie is true!
- The faster and bigger the core the lower its efficiency
- An infinitely fast or infinitely big von Neumann core does no computing!

Von Neumann Computer

- Up to 5 memory transactions per operation: c := a + b has four reads of +, a, b, c and one write of the result to c
- I/O bandwidth is 5c where c is the number of cores in a chip
- Can crash on logical exceptions

Serial Processing

Instruction 1

Data 1

Instruction 2

Instruction 3

Instruction n

Serial Processing

Instruction 1

Instruction 2

Data 1

Instruction 3

Instruction n

Serial Processing

Instruction 1

Instruction 2

Instruction 3

Data 1

Instruction n

Serial Processing

Instruction 1

Instruction 2

Instruction 3

Instruction n

Data 1

Serial Processing

Instruction 1

Data 2

Instruction 2

Instruction 3

Instruction n

Serial Processing

Instruction 1

Instruction 2

Data 2

Instruction 3

Instruction n

Serial Processing

Instruction 1

Instruction 2

Instruction 3

Data 2

Instruction n

Serial Processing

Instruction 1

Instruction 2

Instruction 3

Instruction n

Data 2

Serial Processing

Instruction 1

Data 3

Instruction 2

Instruction 3

Instruction n

Serial Processing

Instruction 1

Instruction 2

Data 3

Instruction 3

Instruction n

Serial Processing

Instruction 1

Instruction 2

Instruction 3

Data 3

Instruction n

Serial Processing

Instruction 1

Instruction 2

Instruction 3

Instruction n

Data 3

Serial Processing

Instruction 1

Data n

Instruction 2

Instruction 3

Instruction n

Serial Processing

Instruction 1

Instruction 2

Data n

Instruction 3

Instruction n

Serial Processing

Instruction 1

Instruction 2

Instruction 3

Data n

Instruction n

Serial Processing

Instruction 1

Instruction 2

Instruction 3

Instruction n

Data n

Slipstream Computer

- Dataflow means travel time is proportional to distance so never stalls
- Dataflow means I/O bandwidth is independent of the number of cores
- Totallity means that if a program compiles it has no logical exceptions so it can crash only on a physical fault
- Totallity means pipelines never break

Slipstream Processing

Instruction 1

Data 1

Instruction 2

Instruction 3

Instruction n

Slipstream Processing

Instruction 1

Data 2

Instruction 2

Data 1

Instruction 3

Instruction n

Slipstream Processing

Instruction 1

Data 3

Instruction 2

Data 2

Instruction 3

Data 1

Instruction n

Slipstream Processing

Instruction 1

Data n

Instruction 2

Data 3

Instruction 3

Data 2

Instruction n

Data 1

Slipstream Processing

Instruction 1

Instruction 2

Data n

Instruction 3

Data 3

Instruction n

Data 2

Slipstream Processing

Instruction 1

Instruction 2

Instruction 3

Data n

Instruction n

Data 3

Slipstream Processing

Instruction 1

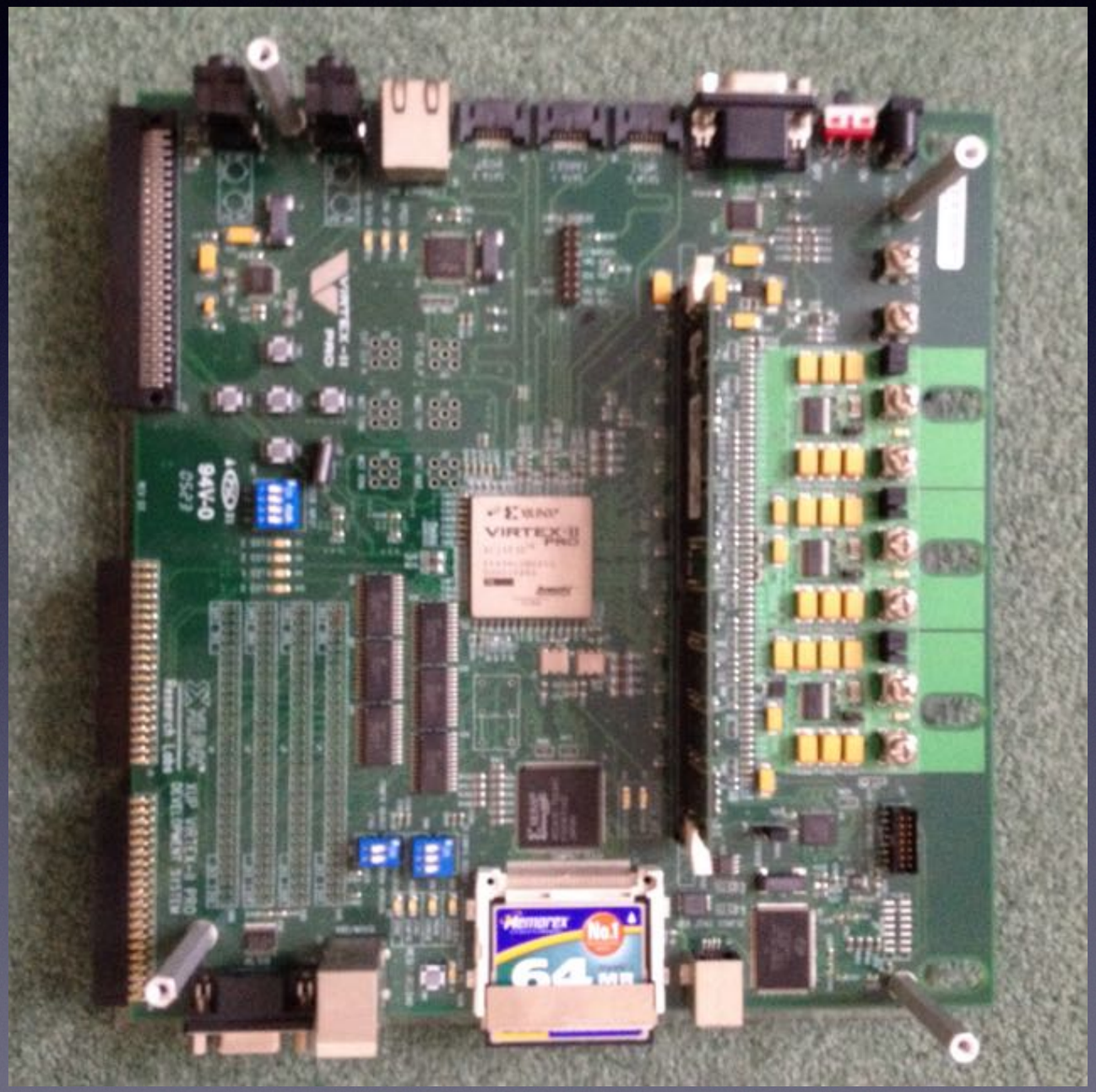
Instruction 2

Instruction 3

Instruction n

Data n

FPGA Prototype



Conclusion

- Transreal arithmetic contains real arithmetic
- Each real number is finite
- There are three non-finite, transreal numbers: negative infinity, nullity, positive infinity
- Transcomputation extends all other computation

Transcomputation

- Google+ Community Transmathematica